1. *CP* VIOLATION IN MESON DECAYS

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The CP transformation combines charge conjugation C with parity P. Under C, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, *e.g.*, $Q \to -Q$ for electromagnetic charge. Under P, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. Thus, for example, a left-handed electron e^-_L is transformed under CP into a right-handed positron, e^+_R.

If CP were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are C- and P-symmetric, and therefore, also CP-symmetric. In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions. The weak interactions, on the other hand, violate C and P in the strongest possible way. For example, the charged W bosons couple to left-handed electrons, e^-_L, and to their CP-conjugate right-handed positrons, e^+_R, but to neither their C-conjugate left-handed positrons, e^+_L, nor their P-conjugate right-handed electrons, e^-_R. While weak interactions violate C and P separately, CP is still preserved in most weak interaction processes. The CP symmetry is, however, violated in certain rare processes, as discovered in neutral K decays in 1964 [1], and observed in recent years in B decays. A K_L meson decays more often to $\pi^- e^+ \overline{\nu}_e$ than to $\pi^+ e^- \nu_e$, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level. The CP-violating effects observed in B decays are larger: the CP asymmetry in $B^0/\overline{B}^0$ meson decays to CP eigenstates like $J/\psi K_S$ is about 0.7 [2,3]. These effects are related to $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ mixing, but CP violation arising solely from decay amplitudes has also been observed, first in $K \to \pi\pi$ decays [4–6], and more recently in various neutral B [7,8] and charged B [9–11] decays. Evidence for CP violation in the decay amplitude at a level higher than 3σ (but still lower than 5σ) has also been achieved in neutral D [12] and B_s [13] decays. CP violation has not yet been observed in the lepton sector.

In addition to parity and to continuous Lorentz transformations, there is one other spacetime operation that could be a symmetry of the interactions: time reversal T, $t \to -t$. Violations of T symmetry have been observed in neutral K decays [14], and are expected as a corollary of CP violation if the combined CPT transformation is a fundamental symmetry of Nature [15]. All observations indicate that CPT is indeed a symmetry of Nature. Furthermore, one cannot build a Lorentz-invariant quantum field theory with a Hermitian Hamiltonian that violates CPT. (At several points in our discussion, we avoid assumptions about CPT, in order to identify cases where evidence for CP violation relies on assumptions about CPT.)

Within the Standard Model, CP symmetry is broken by complex phases in the Yukawa couplings (that is, the couplings of the Higgs scalar to quarks). When all manipulations to remove unphysical phases in this model are exhausted, one finds that there is a single CP-violating parameter [16]. In the basis of mass eigenstates, this single phase appears in the 3×3 unitary matrix that gives the W-boson couplings to an up-type antiquark and a down-type quark. (If the Standard Model is supplemented with Majorana mass terms for the neutrinos, the analogous mixing matrix for leptons has three CP-violating phases.)

The beautifully consistent and economical Standard-Model description of CP violation in terms of Yukawa couplings, known as the Kobayashi-Maskawa (KM) mechanism [16], agrees with all measurements to date. (The measurement of the dimuon asymmetry in semi-leptonic b-hadron decays deviates from the Standard Model prediction by 3.9σ [17]. Pending confirmation, we do not discuss it further in this review.) Furthermore, one can fit the data allowing new physics contributions to loop processes to compete with, or even dominate over, the Standard Model ones [18,19]. Such an analysis provides a model-independent proof that the KM phase is different from zero, and that the matrix of three-generation quark mixing is the dominant source of CP violation in meson decays.

The current level of experimental accuracy and the theoretical uncertainties involved in the interpretation of the various observations leave room, however, for additional subdominant sources of CP violation from new physics. Indeed, almost all extensions of the Standard Model imply that there are such additional sources. Moreover, CP violation is a necessary condition for baryogenesis, the process of dynamically generating the matter-antimatter asymmetry of the Universe [20]. Despite the phenomenological success of the KM mechanism, it fails (by several orders of magnitude) to accommodate the observed asymmetry [21]. This discrepancy strongly suggests that Nature provides additional sources of CP violation beyond the KM mechanism. (The evidence for neutrino masses implies that CP can be violated also in the lepton sector. This situation makes leptogenesis [22], a scenario where CP-violating phases in the Yukawa couplings of the neutrinos play a crucial role in the generation of the baryon asymmetry, a very attractive possibility.) The expectation of new sources motivates the large ongoing experimental effort to find deviations from the predictions of the KM mechanism.

CP violation can be experimentally searched for in a variety of processes, such as meson decays, electric dipole moments of neutrons, electrons and nuclei, and neutrino oscillations. Meson decays probe flavor-changing CP violation. The search for electric dipole moments may find (or constrain) sources of CP violation that, unlike the KM phase, are not related to flavor-changing couplings. Future searches for CP violation in neutrino oscillations might provide further input on leptogenesis.

The present measurements of CP asymmetries provide some of the strongest constraints on the weak couplings of quarks. Future measurements of CP violation in K, D, B, and B_s meson decays will provide additional constraints on the flavor parameters of the Standard Model, and can probe new physics. In this review, we give the formalism and basic physics that are relevant to present and near future measurements of CP violation in meson decays.

Before going into details, we list here the observables where CP violation has been observed at a level above 5σ [23–25]:

- Indirect CP violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays, and in the $K_L \to \pi^+\pi^-e^+e^-$ decay, is given by
$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}\ . \tag{1.1}$$
- Direct CP violation in $K \to \pi\pi$ decays is given by
$$\mathcal{R}e(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}. \tag{1.2}$$

- CP violation in the interference of mixing and decay in the tree-dominated $b \to c\bar{c}s$ transitions, such as $B \to \psi K^0$, is given by (we use K^0 throughout to denote results that combine K_S and K_L modes, but use the sign appropriate to K_S):
$$S_{\psi K^0} = +0.679 \pm 0.020. \tag{1.3}$$
- CP violation in the interference of mixing and decay in various modes related to $b \to q\bar{q}s$ (penguin) transitions is given by
$$S_{\eta' K^0} = +0.59 \pm 0.07, \tag{1.4}$$
$$S_{\phi K^0} = +0.74^{+0.11}_{-0.13}, \tag{1.5}$$
$$S_{f_0 K^0} = +0.69^{+0.10}_{-0.12}, \tag{1.6}$$
$$S_{K^+K^-K_S} = +0.68^{+0.09}_{-0.10}, \tag{1.7}$$
- CP violation in the interference of mixing and decay in the $B \to \pi^+\pi^-$ mode is given by
$$S_{\pi^+\pi^-} = -0.65 \pm 0.07. \tag{1.8}$$
- Direct CP violation in the $B \to \pi^+\pi^-$ mode is given by
$$C_{\pi^+\pi^-} = -0.36 \pm 0.06. \tag{1.9}$$
- CP violation in the interference of mixing and decay in various modes related to $b \to c\bar{c}d$ transitions is given by
$$S_{\psi\pi^0} = -0.93 \pm 0.15, \tag{1.10}$$
$$S_{D^+D^-} = -0.98 \pm 0.17. \tag{1.11}$$
$$S_{D^{*+}D^{*-}} = -0.77 \pm 0.10. \tag{1.12}$$
- Direct CP violation in the $\overline{B}^0 \to K^-\pi^+$ mode is given by
$$\mathcal{A}_{K^\mp\pi^\pm} = -0.087 \pm 0.008. \tag{1.13}$$
- Direct CP violation in $B^\pm \to D_+K^\pm$ decays (D_+ is the CP-even neutral D state) is given by
$$\mathcal{A}_{D_+K^\pm} = +0.19 \pm 0.03. \tag{1.14}$$

1.1. Formalism

The phenomenology of CP violation is superficially different in K, D, B, and B_s decays. This is primarily because each of these systems is governed by a different balance between decay rates, oscillations, and lifetime splitting. However, the underlying mechanisms of CP violation are identical for all pseudoscalar mesons.

In this section, we present a general formalism for, and classification of, CP violation in the decay of a pseudoscalar meson M that might be a charged or neutral K, D, B, or B_s meson. Subsequent sections describe the CP-violating phenomenology, approximations, and alternative formalisms that are specific to each system.

1.1.1. ***Charged- and neutral-meson decays***: We define decay amplitudes of M (which could be charged or neutral) and its CP conjugate $\overline{M}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f\,|\mathcal{H}|\,M\rangle \quad , \quad \overline{A}_f = \langle f\,|\mathcal{H}|\,\overline{M}\rangle \quad ,$$

$$A_{\overline{f}} = \langle \overline{f}\,|\mathcal{H}|\,M\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}\,|\mathcal{H}|\,\overline{M}\rangle \quad , \tag{1.15}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_M and ξ_f that depend on their flavor content, according to

$$CP|M\rangle = e^{+i\xi_M}\,|\overline{M}\rangle \quad , \quad CP|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \; , \tag{1.16}$$

with

$$CP|\overline{M}\rangle = e^{-i\xi_M}\,|M\rangle \quad , \quad CP|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \tag{1.17}$$

so that $(CP)^2 = 1$. The phases ξ_M and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP, \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_M)}\, A_f \; . \tag{1.18}$$

1.1.2. ***Neutral-meson mixing***: A state that is initially a superposition of M^0 and $\overline{M}^0$, say

$$|\psi\,(0)\rangle = a\,(0)\,|M^0\rangle + b\,(0)\,|\overline{M}^0\rangle \; , \tag{1.19}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi\,(t)\rangle = a\,(t)\,|M^0\rangle + b\,(t)\,|\overline{M}^0\rangle + c_1\,(t)\,|f_1\rangle + c_2\,(t)\,|f_2\rangle + \cdots \; . \tag{1.20}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [26]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathbf{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathbf{H}$, can be written in terms of Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ as

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\,\mathbf{\Gamma} \; . \tag{1.21}$$

$\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with $(M^0, \overline{M}^0) \leftrightarrow (M^0, \overline{M}^0)$ transitions via off-shell (dispersive), and on-shell (absorptive) intermediate states, respectively. Diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with the flavor-conserving transitions $M^0 \to M^0$ and $\overline{M}^0 \to \overline{M}^0$, while off-diagonal elements are associated with flavor-changing transitions $M^0 \leftrightarrow \overline{M}^0$.

The eigenvectors of $\mathbf{H}$ have well-defined masses and decay widths. To specify the components of the strong interaction eigenstates, M^0 and $\overline{M}^0$, in the light (M_L) and heavy (M_H) mass eigenstates, we introduce three complex parameters: p, q, and, for the case that both CP and CPT are violated in mixing, z:

$$|M_L\rangle \propto \; p\sqrt{1-z}\,|M^0\rangle + q\sqrt{1+z}\,|\overline{M}^0\rangle$$

$$|M_H\rangle \propto \; p\sqrt{1+z}\,|M^0\rangle - q\sqrt{1-z}\,|\overline{M}^0\rangle \; , \tag{1.22}$$

with the normalization $|q|^2 + |p|^2 = 1$ when $z = 0$. (Another possible choice, which is in standard usage for K mesons, defines the mass eigenstates according to their lifetimes: K_S for the short-lived and K_L for the long-lived state. The K_L is experimentally found to be the heavier state.)

The real and imaginary parts of the eigenvalues $\omega_{L,H}$ corresponding to $|M_{L,H}\rangle$ represent their masses and decay widths, respectively. The mass and width splittings are

$$\begin{aligned}\Delta m \equiv&\ \ m_H - m_L = \mathcal{R}e(\omega_H - \omega_L) \quad , \\ \Delta\Gamma \equiv&\ \ \Gamma_H - \Gamma_L = -2\,\mathcal{I}m(\omega_H - \omega_L) \ .\end{aligned} \tag{1.23}$$

Note that here Δm is positive by definition, while the sign of $\Delta\Gamma$ is to be experimentally determined. The sign of $\Delta\Gamma$ has not yet been established for the B mesons, while $\Delta\Gamma < 0$ is established for K and B_s mesons and $\Delta\Gamma > 0$ is established for D mesons. The Standard Model predicts $\Delta\Gamma < 0$ also for B mesons (for this reason, $\Delta\Gamma = \Gamma_L - \Gamma_H$, which is still a signed quantity, is often used in the B and B_s literature and is the convention used in the PDG experimental summaries).

Solving the eigenvalue problem for $\mathbf{H}$ yields

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}_{12}^* - (i/2)\,\mathbf{\Gamma}_{12}^*}{\mathbf{M}_{12} - (i/2)\,\mathbf{\Gamma}_{12}} \tag{1.24}$$

and

$$z \equiv \frac{\delta m - (i/2)\,\delta\Gamma}{\Delta m - (i/2)\,\Delta\Gamma} \ , \tag{1.25}$$

where

$$\delta m \equiv \mathbf{M}_{11} - \mathbf{M}_{22} \quad , \quad \delta\Gamma \equiv \mathbf{\Gamma}_{11} - \mathbf{\Gamma}_{22} \tag{1.26}$$

are the differences in effective mass and decay-rate expectation values for the strong interaction states M^0 and $\overline{M}^0$.

If either CP or CPT is a symmetry of $\mathbf{H}$ (independently of whether T is conserved or violated), then the values of δm and $\delta\Gamma$ are both zero, and hence $z = 0$. We also find that

$$\omega_H - \omega_L = 2\sqrt{\left(\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}\right)\left(\mathbf{M}_{12}^* - \frac{i}{2}\mathbf{\Gamma}_{12}^*\right)} \ . \tag{1.27}$$

If either CP or T is a symmetry of $\mathbf{H}$ (independently of whether CPT is conserved or violated), then $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_M} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \ , \tag{1.28}$$

where ξ_M is the arbitrary unphysical phase introduced in Eq. (1.17). If, and only if, CP is a symmetry of $\mathbf{H}$ (independently of CPT and T), then both of the above conditions hold, with the result that the mass eigenstates are orthogonal

$$\langle M_H | M_L \rangle = |p|^2 - |q|^2 = 0 \ . \tag{1.29}$$

1.1.3. ***CP-violating observables***: All CP-violating observables in M and $\overline{M}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$, and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $M^0 \leftrightarrow \overline{M}^0$ oscillations, and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

The decay rates of the two neutral K mass eigenstates, K_S and K_L, are different enough ($\Gamma_S/\Gamma_L \sim 500$) that one can, in most cases, actually study their decays independently. For neutral D, B, and B_s mesons, however, values of $\Delta\Gamma/\Gamma$ (where $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$) are relatively small, and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|M^0\rangle$ or $|\overline{M}^0\rangle$ after an elapsed proper time t as $|M^0_{\rm phys}(t)\rangle$ or $|\overline{M}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, but not assuming CPT is a good symmetry, we obtain

$$\begin{aligned} |M^0_{\rm phys}(t)\rangle &= \left(g_+(t) + z\, g_-(t)\right) |M^0\rangle - \sqrt{1-z^2}\,\frac{q}{p}\, g_-(t)\, |\overline{M}^0\rangle, \\ |\overline{M}^0_{\rm phys}(t)\rangle &= \left(g_+(t) - z\, g_-(t)\right) |\overline{M}^0\rangle - \sqrt{1-z^2}\,\frac{p}{q}\, g_-(t)\, |M^0\rangle\,, \end{aligned} \tag{1.30}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right) \tag{1.31}$$

and $z = 0$ if either CPT or CP is conserved.

Defining $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$, and assuming $z = 0$, one obtains the following time-dependent decay rates:

$$\begin{aligned} &\frac{d\Gamma\left[M^0_{\rm phys}(t) \to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \\ &\quad \left(|A_f|^2 + |(q/p)\,\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\,\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &\quad + 2\,\mathcal{R}e((q/p)\,A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)\,A_f^*\overline{A}_f)\sin(x\Gamma t)\,, \end{aligned} \tag{1.32}$$

$$\begin{aligned} &\frac{d\Gamma\left[\overline{M}^0_{\rm phys}(t) \to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \\ &\quad \left(|(p/q)\,A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)\,A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &\quad + 2\,\mathcal{R}e((p/q)\,A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)\,A_f\overline{A}_f^*)\sin(x\Gamma t)\,, \end{aligned} \tag{1.33}$$

where $\mathcal{N}_f$ is a common, time-independent, normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (1.32, 1.33). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $M \leftrightarrow \overline{M}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (1.32, 1.33) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

When neutral pseudoscalar mesons are produced coherently in pairs from the decay of a vector resonance, $V \to M^0\overline{M}^0$ (for example, $\Upsilon(4S) \to B^0\overline{B}^0$ or $\phi \to K^0\overline{K}^0$), the time-dependence of their subsequent decays to final states f_1 and f_2 has a similar form to

Eqs. (1.32, 1.33):

$$\frac{d\Gamma\left[V_{\rm phys}\left(t_1,t_2\right)\to f_1 f_2\right]/d\left(\Delta t\right)}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1 f_2}} =$$

$$\left(|a_+|^2+|a_-|^2\right)\cosh\left(y\Gamma\Delta t\right)+\left(|a_+|^2-|a_-|^2\right)\cos\left(x\Gamma\Delta t\right)$$

$$-\,2\,\mathcal{R}e(a_+^* a_-)\sinh\left(y\Gamma\Delta t\right)+2\,\mathcal{I}m(a_+^* a_-)\sin\left(x\Gamma\Delta t\right)\ , \tag{1.34}$$

where $\Delta t \equiv t_2 - t_1$ is the difference in the production times, t_1 and t_2, of f_1 and f_2, respectively, and the dependence on the average decay time and on decay angles has been integrated out. The coefficients in Eq. (1.34) are determined by the amplitudes for no net oscillation from $t_1 \to t_2$, $\overline{A}_{f_1} A_{f_2}$, and $A_{f_1}\overline{A}_{f_2}$, and for a net oscillation, $(q/p)\overline{A}_{f_1}\overline{A}_{f_2}$ and $(p/q)A_{f_1}A_{f_2}$, via

$$a_+ \equiv\ \overline{A}_{f_1}A_{f_2} - A_{f_1}\overline{A}_{f_2}\ , \tag{1.35}$$

$$a_- \equiv\ -\sqrt{1-z^2}\left(\frac{q}{p}\overline{A}_{f_1}\overline{A}_{f_2} - \frac{p}{q}A_{f_1}A_{f_2}\right) + z\left(\overline{A}_{f_1}A_{f_2} + A_{f_1}\overline{A}_{f_2}\right)\ .$$

Assuming CPT conservation, $z=0$, and identifying $\Delta t \to t$ and $f_2 \to f$, we find that Eqs. (1.34) and (1.35) reduce to Eq. (1.32) with $A_{f_1}=0$, $\overline{A}_{f_1}=1$, or to Eq. (1.33) with $\overline{A}_{f_1}=0$, $A_{f_1}=1$. Indeed, such a situation plays an important role in experiments. Final states f_1 with $A_{f_1}=0$ or $\overline{A}_{f_1}=0$ are called tagging states, because they identify the decaying pseudoscalar meson as, respectively, $\overline{M}^0$ or M^0. Before one of M^0 or $\overline{M}^0$ decays, they evolve in phase, so that there is always one M^0 and one $\overline{M}^0$ present. A tagging decay of one meson sets the clock for the time evolution of the other: it starts at t_1 as purely M^0 or $\overline{M}^0$, with time evolution that depends only on $t_2 - t_1$.

When f_1 is a state that both M^0 and $\overline{M}^0$ can decay into, then Eq. (1.34) contains interference terms proportional to $A_{f_1}\overline{A}_{f_1} \neq 0$ that are not present in Eqs. (1.32, 1.33). Even when f_1 is dominantly produced by M^0 decays rather than $\overline{M}^0$ decays, or vice versa, $A_{f_1}\overline{A}_{f_1}$ can be non-zero owing to doubly-CKM-suppressed decays (with amplitudes suppressed by at least two powers of λ relative to the dominant amplitude, in the language of Section 12.3), and these terms should be considered for precision studies of CP violation in coherent $V \to M^0\overline{M}^0$ decays [27].

1.1.4. ***Classification of CP-violating effects***: We distinguish three types of CP-violating effects in meson decays:

I. CP violation in decay is defined by

$$|\overline{A}_{\overline{f}}/A_f| \neq 1\ . \tag{1.36}$$

In charged meson decays, where mixing effects are absent, this is the only possible source of CP asymmetries:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma\left(M^-\to f^-\right)-\Gamma\left(M^+\to f^+\right)}{\Gamma\left(M^-\to f^-\right)+\Gamma\left(M^+\to f^+\right)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2-1}{|\overline{A}_{f^-}/A_{f^+}|^2+1}\ . \tag{1.37}$$

II. CP (and T) violation in mixing is defined by

$$|q/p| \neq 1\ . \tag{1.38}$$

In charged-current semileptonic neutral meson decays $M,\overline{M} \to \ell^\pm X$ (taking $|A_{\ell^+X}| = |\overline{A}_{\ell^-X}|$ and $A_{\ell^-X} = \overline{A}_{\ell^+X} = 0$, as is the case in the Standard Model, to lowest order in G_F, and in most of its reasonable extensions), this is the only source of CP violation, and can be measured via the asymmetry of "wrong-sign" decays induced by oscillations:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\right] - d\Gamma/dt\left[M^0_{\rm phys}(t) \to \ell^- X\right]}{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\right] + d\Gamma/dt\left[M^0_{\rm phys}(t) \to \ell^- X\right]} = \frac{1-|q/p|^4}{1+|q/p|^4}. \tag{1.39}$$

Note that this asymmetry of time-dependent decay rates is actually time-independent.

III. CP violation in interference between a decay without mixing, $M^0 \to f$, and a decay with mixing, $M^0 \to \overline{M}^0 \to f$ (such an effect occurs only in decays to final states that are common to M^0 and $\overline{M}^0$, including all CP eigenstates), is defined by

$$\mathcal{I}m(\lambda_f) \neq 0\,, \tag{1.40}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}\,. \tag{1.41}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to f_{CP}\right] - d\Gamma/dt\left[M^0_{\rm phys}(t) \to f_{CP}\right]}{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to f_{CP}\right] + d\Gamma/dt\left[M^0_{\rm phys}(t) \to f_{CP}\right]}\,. \tag{1.42}$$

If $\Delta\Gamma = 0$ and $|q/p| = 1$, as expected to a good approximation for B_d mesons, but not for K and B_s mesons, then $\mathcal{A}_{f_{CP}}$ has a particularly simple form (see Eq. (1.86), below). If, in addition, the decay amplitudes fulfill $|\overline{A}_{f_{CP}}| = |A_{f_{CP}}|$, the interference between decays with and without mixing is the only source of the asymmetry and $\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_{f_{CP}})\sin(x\Gamma t)$.

Examples of these three types of CP violation will be given in Sections 1.4, 1.5, and 1.6.

1.2. Theoretical Interpretation: General Considerations

Consider the $M \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{M} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus, their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons, and hence, are often called "weak phases." The weak phase of any single term is convention-dependent. However, the difference between the weak phases in two different terms in A_f is convention-independent. A second type of phase can appear in scattering or decay amplitudes, even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions; hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (1.18). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f = & \ |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} = & \ |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{1.43}$$

Similarly, for neutral meson decays, it is useful to write

$$\mathbf{M}_{12} = |\mathbf{M}_{12}| e^{i\phi_M} \quad , \quad \mathbf{\Gamma}_{12} = |\mathbf{\Gamma}_{12}| e^{i\phi_\Gamma} \ . \tag{1.44}$$

Each of the phases appearing in Eqs. (1.43, 1.44) is convention-dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$, and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

It is now straightforward to evaluate the various asymmetries in terms of the theoretical parameters introduced here. We will do so with approximations that are often relevant to the most interesting measured asymmetries.

1. The CP asymmetry in charged meson decays [Eq. (1.37)] is given by

$$\mathcal{A}_{f^\pm} = -\frac{2|a_1 a_2| \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)}. \tag{1.45}$$

The quantity of most interest to theory is the weak phase difference $\phi_2 - \phi_1$. Its extraction from the asymmetry requires, however, that the amplitude ratio $|a_2/a_1|$ and the strong phase difference $\delta_2 - \delta_1$ are known. Both quantities depend on non-perturbative hadronic parameters that are difficult to calculate.

2. In the approximation that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| \ll 1$ (valid for B and B_s mesons), the CP asymmetry in semileptonic neutral-meson decays [Eq. (1.39)] is given by

$$\mathcal{A}_{\rm SL} = -\left|\frac{\mathbf{\Gamma}_{12}}{\mathbf{M}_{12}}\right| \sin(\phi_M - \phi_\Gamma) \, . \tag{1.46}$$

The quantity of most interest to theory is the weak phase $\phi_M - \phi_\Gamma$. Its extraction from the asymmetry requires, however, that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}|$ is known. This quantity depends on long-distance physics that is difficult to calculate.

3. In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f + \phi_f)}$, and that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| = 0$, we obtain $|\lambda_f| = 1$, and the CP asymmetries in decays to a final CP eigenstate f [Eq. (1.42)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f) \ \sin(\Delta m t) \quad \text{with} \quad \mathcal{I}m(\lambda_f) = \eta_f \sin\left(\phi_M + 2\phi_f\right) . \tag{1.47}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

The discussion above allows us to introduce another classification of CP-violating effects:

1. *Indirect CP violation* is consistent with taking $\phi_M \neq 0$ and setting all other CP violating phases to zero. CP violation in mixing (type II) belongs to this class.
2. *Direct CP violation* cannot be accounted for by just $\phi_M \neq 0$. CP violation in decay (type I) belongs to this class.

As concerns type III CP violation, observing $\eta_{f_1}\mathcal{I}m(\lambda_{f_1}) \neq \eta_{f_2}\mathcal{I}m(\lambda_{f_2})$ (for the same decaying meson and two different final CP eigenstates f_1 and f_2) would establish direct CP violation. The significance of this classification is related to theory. In superweak models [28], CP violation appears only in diagrams that contribute to $\mathbf{M}_{12}$, hence they predict that there is no direct CP violation. In most models and, in particular, in the Standard Model, CP violation is both direct and indirect. The experimental observation of $\epsilon' \neq 0$ (see Section 1.4) excluded the superweak scenario.

1.3. Theoretical Interpretation: The KM Mechanism

Of all the Standard Model quark parameters, only the Kobayashi-Maskawa (KM) phase is CP-violating. Having a single source of CP violation, the Standard Model is very predictive for CP asymmetries: some vanish, and those that do not are correlated.

To be precise, CP could be violated also by strong interactions. The experimental upper bound on the electric-dipole moment of the neutron implies, however, that $\theta_{\rm QCD}$, the non-perturbative parameter that determines the strength of this type of CP violation, is tiny, if not zero. (The smallness of $\theta_{\rm QCD}$ constitutes a theoretical puzzle, known as 'the strong CP problem.') In particular, it is irrelevant to our discussion of meson decays.

The charged current interactions (that is, the $W^\pm$ interactions) for quarks are given by

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\, \overline{u_{Li}}\, \gamma^\mu\, (V_{\rm CKM})_{ij}\, d_{Lj}\, W_\mu^+ + \text{h.c.} \tag{1.48}$$

Here $i,j = 1,2,3$ are generation numbers. The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks is a 3×3 unitary matrix [29]. Ordering the quarks by their masses, *i.e.*, $(u_1,u_2,u_3) \to (u,c,t)$ and $(d_1,d_2,d_3) \to (d,s,b)$, the elements of $V_{\rm CKM}$ are written as follows:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1.49}$$

While a general 3×3 unitary matrix depends on three real angles and six phases, the freedom to redefine the phases of the quark mass eigenstates can be used to remove five of the phases, leaving a single physical phase, the Kobayashi-Maskawa phase, that is responsible for all CP violation in meson decays in the Standard Model.

The fact that one can parametrize $V_{\rm CKM}$ by three real and only one imaginary physical parameters can be made manifest by choosing an explicit parametrization. The Wolfenstein parametrization [30,31] is particularly useful:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5\left[1-2(\rho+i\eta)\right] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4\left(1+4A^2\right) & A\lambda^2 \\ A\lambda^3\left[1-\left(1-\frac{1}{2}\lambda^2\right)(\rho+i\eta)\right] & -A\lambda^2+\frac{1}{2}A\lambda^4\left[1-2(\rho+i\eta)\right] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{1.50}$$

Here $\lambda \approx 0.23$ (not to be confused with λ_f), the sine of the Cabibbo angle, plays the role of an expansion parameter, and η represents the CP-violating phase. Terms of $\mathcal{O}(\lambda^6)$ were neglected.

The unitarity of the CKM matrix, $(VV^\dagger)_{ij} = (V^\dagger V)_{ij} = \delta_{ij}$, leads to twelve distinct complex relations among the matrix elements. The six relations with $i \neq j$ can be represented geometrically as triangles in the complex plane. Two of these,

$$\begin{aligned} V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \\ V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* &= 0\,, \end{aligned}$$

have terms of equal order, $\mathcal{O}(A\lambda^3)$, and so have corresponding triangles whose interior angles are all $\mathcal{O}(1)$ physical quantities that can be independently measured. The angles of the first triangle (see Fig. 1.1) are given by

$$\begin{aligned} \alpha \equiv \varphi_2 \equiv &\ \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1-\rho-i\eta}{\rho+i\eta}\right), \\ \beta \equiv \varphi_1 \equiv &\ \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1-\rho-i\eta}\right), \\ \gamma \equiv \varphi_3 \equiv &\ \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg\left(\rho+i\eta\right). \end{aligned} \tag{1.51}$$

The angles of the second triangle are equal to (α, β, γ) up to corrections of $\mathcal{O}(\lambda^2)$. The notations (α, β, γ) and $(\varphi_1, \varphi_2, \varphi_3)$ are both in common usage but, for convenience, we only use the first convention in the following.

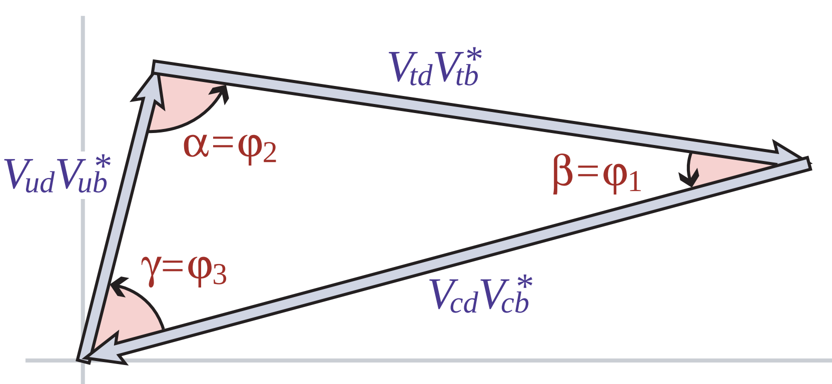


Figure 1.1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Another relation that can be represented as a triangle,

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0\,, \tag{1.52}$$

and, in particular, its small angle, of $\mathcal{O}(\lambda^2)$,

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right), \tag{1.53}$$

is convenient for analyzing CP violation in the B_s sector.

All unitarity triangles have the same area, commonly denoted by $J/2$ [32]. If CP is violated, J is different from zero and can be taken as the single CP-violating parameter. In the Wolfenstein parametrization of Eq. (1.50), $J \simeq \lambda^6 A^2 \eta$.

1.4. *K* Decays

CP violation was discovered in $K \to \pi\pi$ decays in 1964 [1]. The same mode provided the first evidence for direct CP violation [4–6].

The decay amplitudes actually measured in neutral K decays refer to the mass eigenstates K_L and K_S, rather than to the K and $\overline{K}$ states referred to in Eq. (1.15). The final $\pi^+\pi^-$ and $\pi^0\pi^0$ states are CP-even. In the CP limit, $K_S(K_L)$ would be CP-even (odd), and therefore would (would not) decay to two pions. We define CP-violating amplitude ratios for two-pion final states,

$$\eta_{00} \equiv \frac{\langle\pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle\pi^0\pi^0|\mathcal{H}|K_S\rangle} \quad , \quad \eta_{+-} \equiv \frac{\langle\pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle\pi^+\pi^-|\mathcal{H}|K_S\rangle} \ . \tag{1.54}$$

Another important observable is the asymmetry of time-integrated semileptonic decay rates:

$$\delta_L \equiv \frac{\Gamma\left(K_L \to \ell^+\nu_\ell\pi^-\right) - \Gamma\left(K_L \to \ell^-\overline{\nu}_\ell\pi^+\right)}{\Gamma\left(K_L \to \ell^+\nu_\ell\pi^-\right) + \Gamma\left(K_L \to \ell^-\overline{\nu}_\ell\pi^+\right)} \ . \tag{1.55}$$

CP violation has been observed as an appearance of K_L decays to two-pion final states [23],

$$|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3} \qquad |\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3} \tag{1.56}$$

$$|\eta_{00}/\eta_{+-}| = 0.9951 \pm 0.0008 \ , \tag{1.57}$$

where the phase ϕ_{ij} of the amplitude ratio η_{ij} has been determined both assuming CPT invariance:

$$\phi_{00} = (43.52 \pm 0.06)^\circ \ , \qquad \phi_{+-} = (43.51 \pm 0.05)^\circ \ , \tag{1.58}$$

and without assuming CPT invariance:

$$\phi_{00} = (43.7 \pm 0.8)^\circ \ , \qquad \phi_{+-} = (43.4 \pm 0.7)^\circ \ . \tag{1.59}$$

CP violation has also been observed in semileptonic K_L decays [23]

$$\delta_L = (3.32 \pm 0.06) \times 10^{-3} \ , \tag{1.60}$$

where δ_L is a weighted average of muon and electron measurements, as well as in K_L decays to $\pi^+\pi^-\gamma$ and $\pi^+\pi^-e^+e^-$ [23]. CP violation in $K \to 3\pi$ decays has not yet been observed [23,33].

Historically, CP violation in neutral K decays has been described in terms of parameters ϵ and ϵ'. The observables η_{00}, η_{+-}, and δ_L are related to these parameters, and to those of Section 1.1, by

$$\begin{aligned}
\eta_{00} = &\ \frac{1-\lambda_{\pi^0\pi^0}}{1+\lambda_{\pi^0\pi^0}} &= &\ \epsilon - 2\epsilon' \ , \\
\eta_{+-} = &\ \frac{1-\lambda_{\pi^+\pi^-}}{1+\lambda_{\pi^+\pi^-}} &= &\ \epsilon + \epsilon' \ , \\
\delta_L = &\ \frac{1-|q/p|^2}{1+|q/p|^2} &= &\ \frac{2\mathcal{R}e(\epsilon)}{1+|\epsilon|^2} \ ,
\end{aligned} \tag{1.61}$$

where, in the last line, we have assumed that $\left|A_{\ell^+\nu_\ell\pi^-}\right| = \left|\overline{A}_{\ell^-\overline{\nu}_\ell\pi^+}\right|$ and $\left|A_{\ell^-\overline{\nu}_\ell\pi^+}\right| = \left|\overline{A}_{\ell^+\nu_\ell\pi^-}\right| = 0$. (The convention-dependent parameter $\tilde{\epsilon} \equiv (1-q/p)/(1+q/p)$, sometimes used in the literature, is, in general, different from ϵ but yields a similar expression, $\delta_L = 2\mathcal{R}e(\tilde{\epsilon})/(1+|\tilde{\epsilon}|^2)$.) A fit to the $K \to \pi\pi$ data yields [23]

$$\begin{aligned} |\epsilon| = &\ (2.228 \pm 0.011) \times 10^{-3} , \\ \mathcal{R}e(\epsilon'/\epsilon) = &\ (1.65 \pm 0.26) \times 10^{-3} . \end{aligned} \tag{1.62}$$

In discussing two-pion final states, it is useful to express the amplitudes $A_{\pi^0\pi^0}$ and $A_{\pi^+\pi^-}$ in terms of their isospin components via

$$\begin{aligned} A_{\pi^0\pi^0} = &\ \sqrt{\frac{1}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} - \sqrt{\frac{2}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)}, \\ A_{\pi^+\pi^-} = &\ \sqrt{\frac{2}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} + \sqrt{\frac{1}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)} , \end{aligned} \tag{1.63}$$

where we parameterize the amplitude $A_I(\overline{A}_I)$ for $K^0(\overline{K}^0)$ decay into two pions with total isospin $I = 0$ or 2 as

$$\begin{aligned} A_I \equiv &\ \langle(\pi\pi)_I\,|\mathcal{H}|\,K^0\rangle = |A_I|\,e^{i(\delta_I+\phi_I)} , \\ \overline{A}_I \equiv &\ \langle(\pi\pi)_I\,|\mathcal{H}|\,\overline{K}^0\rangle = |A_I|\,e^{i(\delta_I-\phi_I)} . \end{aligned} \tag{1.64}$$

The smallness of $|\eta_{00}|$ and $|\eta_{+-}|$ allows us to approximate

$$\epsilon \simeq \frac{1}{2}\left(1 - \lambda_{(\pi\pi)_{I=0}}\right), \qquad \epsilon' \simeq \frac{1}{6}\left(\lambda_{\pi^0\pi^0} - \lambda_{\pi^+\pi^-}\right) . \tag{1.65}$$

The parameter ϵ represents indirect CP violation, while ϵ' parameterizes direct CP violation: $\mathcal{R}e(\epsilon')$ measures CP violation in decay (type I), $\mathcal{R}e(\epsilon)$ measures CP violation in mixing (type II), and $\mathcal{I}m(\epsilon)$ and $\mathcal{I}m(\epsilon')$ measure the interference between decays with and without mixing (type III).

The following expressions for ϵ and ϵ' are useful for theoretical evaluations:

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\mathcal{I}m(\mathbf{M}_{12})}{\Delta m}, \qquad \epsilon' = \frac{i}{\sqrt{2}}\left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)} \sin\left(\phi_2 - \phi_0\right) . \tag{1.66}$$

The expression for ϵ is only valid in a phase convention where $\phi_2 = 0$, corresponding to a real $V_{ud}V^*_{us}$, and in the approximation that also $\phi_0 = 0$. The phase of ϵ, $\arg(\epsilon) \approx \arctan(-2\Delta m/\Delta\Gamma)$, is independent of the electroweak model and is experimentally determined to be about $\pi/4$. The calculation of ϵ benefits from the fact that $\mathcal{I}m(\mathbf{M}_{12})$ is dominated by short distance physics. Consequently, the main source of uncertainty in theoretical interpretations of ϵ are the values of matrix elements, such as $\langle K^0\,|(\overline{s}d)_{V-A}(\overline{s}d)_{V-A}|\,\overline{K}^0\rangle$. The expression for ϵ' is valid to first order in $|A_2/A_0| \sim 1/20$. The phase of ϵ' is experimentally determined, $\pi/2 + \delta_2 - \delta_0 \approx \pi/4$, and is independent of the electroweak model. Note that, accidentally, ϵ'/ϵ is real to a good approximation.

A future measurement of much interest is that of CP violation in the rare $K \to \pi\nu\overline{\nu}$ decays. The signal for CP violation is simply observing the $K_L \to \pi^0\nu\overline{\nu}$ decay. The effect here is that of interference between decays with and without mixing (type III) [34]:

$$\frac{\Gamma\left(K_L \to \pi^0\nu\overline{\nu}\right)}{\Gamma\left(K^+ \to \pi^+\nu\overline{\nu}\right)} = \frac{1}{2}\left[1 + |\lambda_{\pi\nu\overline{\nu}}|^2 - 2\,\mathcal{R}e(\lambda_{\pi\nu\overline{\nu}})\right] \simeq 1 - \mathcal{R}e(\lambda_{\pi\nu\overline{\nu}}), \tag{1.67}$$

where in the last equation we neglect CP violation in decay and in mixing (expected, model-independently, to be of order 10^{-5} and 10^{-3}, respectively). Such a measurement would be experimentally very challenging and theoretically very rewarding [35]. Similar to the CP asymmetry in $B \to J/\psi K_S$, the CP violation in $K \to \pi\nu\overline{\nu}$ decay is predicted to be large (that is, the ratio in Eq. (1.67) is neither CKM- nor loop-suppressed) and can be very cleanly interpreted.

Within the Standard Model, the $K_L \to \pi^0\nu\overline{\nu}$ decay is dominated by an intermediate top quark contribution and, consequently, can be interpreted in terms of CKM parameters [36]. (For the charged mode, $K^+ \to \pi^+\nu\overline{\nu}$, the contribution from an intermediate charm quark is not negligible, and constitutes a source of hadronic uncertainty.) In particular, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ provides a theoretically clean way to determine the Wolfenstein parameter η [37]:

$$\mathcal{B}\left(K_L \to \pi^0\nu\overline{\nu}\right) = \kappa_L \left[X\left(m_t^2/m_W^2\right)\right]^2 A^4\eta^2, \tag{1.68}$$

where $\kappa_L \sim 2 \times 10^{-10}$ incorporates the value of the four-fermion matrix element which is deduced, using isospin relations, from $\mathcal{B}(K^+ \to \pi^0 e^+\nu)$, and $X(m_t^2/m_W^2)$ is a known function of the top mass.

1.5. *D* Decays

Evidence for D^0–$\overline{D}^0$ mixing has been obtained in recent years [38–40]. The experimental constraints read [25,41] $x \equiv \Delta m/\Gamma = 0.0063 \pm 0.0019$ and $y \equiv \Delta\Gamma/(2\Gamma) = 0.0075 \pm 0.0012$. Long-distance contributions make it difficult to calculate the Standard Model prediction for the D^0–$\overline{D}^0$ mixing parameters. Therefore, the goal of the search for D^0–$\overline{D}^0$ mixing is not to constrain the CKM parameters, but rather to probe new physics. Here CP violation plays an important role. Within the Standard Model, the CP-violating effects are predicted to be small, since the mixing and the relevant decays are described, to an excellent approximation, by physics of the first two generations. The expectation is that the Standard Model size of CP violation in D decays is of $\mathcal{O}(10^{-3})$ or less, but theoretical work is ongoing to understand whether QCD effects can significantly enhance it. At present, the most sensitive searches involve the $D \to K^+K^-$, $D \to \pi^+\pi^-$ and $D \to K^\pm\pi^\mp$ modes.

The neutral D mesons decay via a singly-Cabibbo-suppressed transition to the CP eigenstates K^+K^- and $\pi^+\pi^-$. These decays are dominated by Standard-Model tree diagrams. Thus, we can write, for $f = K^+K^-$ or $\pi^+\pi^-$,

$$\begin{aligned} A_f &= A_f^T e^{+i\phi_f^T}\left[1 + r_f e^{i(\delta_f+\phi_f)}\right] , \\ \bar{A}_f &= A_f^T e^{-i\phi_f^T}\left[1 + r_f e^{i(\delta_f-\phi_f)}\right] , \end{aligned} \tag{1.69}$$

where $A_f^T e^{\pm i\phi_f^T}$ is the SM tree level contribution, ϕ_f^T and ϕ_f are weak, CP violating phases, δ_f is a strong phase, and r_f is the ratio between a subleading ($r_f \ll 1$) contribution with a weak phase different from ϕ_f^T and the SM tree level contribution. Neglecting r_f, λ_f is universal, and we can define a phase ϕ_D via

$$\lambda_f \equiv -|q/p|e^{i\phi_D}. \tag{1.70}$$

(In the limit of CP conservation, choosing $\phi_D = 0$ is equivalent to defining the mass eigenstates by their CP eigenvalue: $|D_\mp\rangle =$

$p|D^0\rangle \pm q|\overline{D}^0\rangle$, with $D_-(D_+)$ being the CP-odd (CP-even) state; that is, the state that does not (does) decay into K^+K^-.)

We define the time integrated CP asymmetry for a final CP eigenstate f as follows:

$$a_f \equiv \frac{\int_0^\infty \Gamma\left(D^0_{\rm phys}(t) \to f\right) dt - \int_0^\infty \Gamma\left(\overline{D}^0_{\rm phys}(t) \to f\right) dt}{\int_0^\infty \Gamma\left(D^0_{\rm phys}(t) \to f\right) dt + \int_0^\infty \Gamma\left(\overline{D}^0_{\rm phys}(t) \to f\right) dt}. \quad (1.71)$$

(This expression corresponds to the D meson being tagged at production, hence the integration goes from 0 to $+\infty$; measurements are also possible with $\psi(3770) \to D\bar{D}$, in which case the integration goes from $-\infty$ to $+\infty$.) We take $x, y, r_f \ll 1$ and expand to leading order in these parameters. Then, we can separate the contribution to a_f to three parts [42],

$$a_f = a_f^d + a_f^m + a_f^i, \quad (1.72)$$

with the following underlying mechanisms:

1. a_f^d signals CP violation in decay (similar to Eq. (1.37)):
$$a_f^d = 2r_f \sin\phi_f \sin\delta_f. \quad (1.73)$$
2. a_f^m signals CP violation in mixing (similar to Eq. (1.46)). With our approximations, it is universal:
$$a^m = -\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\phi_D. \quad (1.74)$$
3. a_f^i signals CP violation in the interference of mixing and decay (similar to Eq. (1.47)). With our approximations, it is universal:
$$a^i = \frac{x}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\phi_D. \quad (1.75)$$

One can isolate the effects of direct CP violation by taking the difference between the CP asymmetries in the K^+K^- and $\pi^+\pi^-$ modes:

$$\Delta a_{CP} \equiv a_{K^+K^-} - a_{\pi^+\pi^-} = a^d_{K^+K^-} - a^d_{\pi^+\pi^-}, \quad (1.76)$$

where we neglected a residual, experiment-dependent, contribution from indirect CP violation due to the fact that there is a time dependent acceptance function that can be different for the K^+K^- and $\pi^+\pi^-$ channels. Recently, evidence for such direct CP violation has been obtained [25]:

$$a^d_{K^+K^-} - a^d_{\pi^+\pi^-} = (-6.4 \pm 1.8) \times 10^{-3}. \quad (1.77)$$

One can also isolate the effects of indirect CP violation in the following way. Consider the time dependent decay rates in Eq. (1.32) and Eq. (1.33). The mixing processes modify the time dependence from a pure exponential. However, given the small values of x and y, the time dependences can be recast, to a good approximation, into purely exponential form, but with modified decay-rate parameters [43]:

$$\begin{aligned}\Gamma_{D^0\to K^+K^-} &= \Gamma \times \left[1 + |q/p|\left(y\cos\phi_D - x\sin\phi_D\right)\right], \\ \Gamma_{\overline{D}^0\to K^+K^-} &= \Gamma \times \left[1 + |p/q|\left(y\cos\phi_D + x\sin\phi_D\right)\right]. \quad (1.78)\end{aligned}$$

One can define CP-conserving and CP-violating combinations of these two observables (normalized to the true width Γ):

$$y_{CP} \equiv \frac{\Gamma_{\overline{D}^0\to K^+K^-} + \Gamma_{D^0\to K^+K^-}}{2\Gamma} - 1$$

$$
\begin{aligned}
&= (y/2)\left(|q/p|+|p/q|\right)\cos\phi_D-(x/2)\left(|q/p|-|p/q|\right)\sin\phi_D\ ,\\
A_\Gamma &\equiv \frac{\Gamma_{D^0\to K^+K^-}-\Gamma_{\overline{D}^0\to K^+K^-}}{2\Gamma}\\
&= -\left(a^m+a^i\right)\ .
\end{aligned}
\tag{1.79}
$$

In the limit of CP conservation (and, in particular, within the Standard Model), $y_{CP}=(\Gamma_+-\Gamma_-)/2\Gamma$ (where $\Gamma_+(\Gamma_-)$ is the decay width of the CP-even (-odd) mass eigenstate) and $A_\Gamma=0$. Indeed, present measurements imply that CP violation is small [25],

$$
\begin{aligned}
y_{CP} &= +(1.06\pm0.21)\times10^{-2},\\
A_\Gamma &= +(0.03\pm0.23)\times10^{-2}.
\end{aligned}
$$

The $K^\pm\pi^\mp$ states are not CP eigenstates, but they are still common final states for D^0 and $\overline{D}^0$ decays. Since $D^0(\overline{D}^0)\to K^-\pi^+$ is a Cabibbo-favored (doubly-Cabibbo-suppressed) process, these processes are particularly sensitive to x and/or $y=\mathcal{O}(\lambda^2)$. Taking into account that $\left|\lambda_{K^-\pi^+}\right|,\left|\lambda^{-1}_{K^+\pi^-}\right|\ll 1$ and $x,y\ll1$, assuming that there is no direct CP violation (these are Standard Model tree-level decays dominated by a single weak phase, and there is no contribution from penguin-like and chromomagnetic operators), and expanding the time-dependent rates for $xt,yt\lesssim\Gamma^{-1}$, one obtains

$$
\begin{aligned}
\Gamma\left[D^0_{\rm phys}(t)\to K^+\pi^-\right] &= e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2\\
\times\quad&\left[r_d^2+r_d\left|\frac{q}{p}\right|\left(y'\cos\phi_D-x'\sin\phi_D\right)\Gamma t+\left|\frac{q}{p}\right|^2\frac{y^2+x^2}{4}(\Gamma t)^2\right]\ ,\\
\Gamma\left[\overline{D}^0_{\rm phys}(t)\to K^-\pi^+\right] &= e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2\\
\times\quad&\left[r_d^2+r_d\left|\frac{p}{q}\right|\left(y'\cos\phi_D+x'\sin\phi_D\right)\Gamma t+\left|\frac{p}{q}\right|^2\frac{y^2+x^2}{4}(\Gamma t)^2\right]\ ,
\end{aligned}
\tag{1.80}
$$

where

$$
\begin{aligned}
y' &\equiv y\cos\delta-x\sin\delta\ ,\\
x' &\equiv x\cos\delta+y\sin\delta\ .
\end{aligned}
\tag{1.81}
$$

The weak phase ϕ_D is the same as that of Eq. (1.70) (a consequence of neglecting direct CP violation), δ is a strong-phase difference for these processes, and $r_d=\mathcal{O}(\tan^2\theta_c)$ is the amplitude ratio, $r_d=\left|\overline{A}_{K^-\pi^+}/A_{K^-\pi^+}\right|=\left|A_{K^+\pi^-}/\overline{A}_{K^+\pi^-}\right|$, that is, $\lambda_{K^-\pi^+}=r_d|q/p|e^{-i(\delta-\phi_D)}$ and $\lambda^{-1}_{K^+\pi^-}=r_d|p/q|e^{-i(\delta+\phi_D)}$. By fitting to the six coefficients of the various time-dependences, one can extract r_d, $|q/p|$, (x^2+y^2), $y'\cos\phi_D$, and $x'\sin\phi_D$. In particular, finding CP violation ($|q/p|\neq1$ and/or $\sin\phi_D\neq0$) at a level much higher than 10^{-3} would constitute evidence for new physics.

A fit to all data [25] yields no evidence for indirect CP violation:

$$
\begin{aligned}
1-|q/p| &= +0.12\pm0.17,\\
\phi_D &= -0.18\pm0.16.
\end{aligned}
$$

More details on various theoretical and experimental aspects of $D^0-\overline{D}^0$ mixing can be found in Ref. [44].

1.6. B and B_s Decays

The upper bound on the CP asymmetry in semileptonic B decays [24] implies that CP violation in $B^0 - \overline{B}^0$ mixing is a small effect (we use $\mathcal{A}_{\rm SL}/2 \approx 1 - |q/p|$, see Eq. (1.39)):

$$\mathcal{A}^d_{\rm SL} = (-3.3 \pm 3.3) \times 10^{-3} \quad \Longrightarrow \quad |q/p| = 1.0017 \pm 0.0017. \tag{1.82}$$

The Standard Model prediction is

$$\mathcal{A}^d_{\rm SL} = \mathcal{O}\left[\left(m_c^2/m_t^2\right)\sin\beta\right] \lesssim 0.001. \tag{1.83}$$

In models where $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately real, such as the Standard Model, an upper bound on $\Delta\Gamma/\Delta m \approx \mathcal{R}e(\mathbf{\Gamma}_{12}/\mathbf{M}_{12})$ provides yet another upper bound on the deviation of $|q/p|$ from one. This constraint does not hold if $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately imaginary. (An alternative parameterization uses $q/p = (1 - \tilde{\epsilon}_B)/(1 + \tilde{\epsilon}_B)$, leading to $\mathcal{A}_{\rm SL} \simeq 4\mathcal{R}e(\tilde{\epsilon}_B)$.)

The small deviation (less than one percent) of $|q/p|$ from 1 implies that, at the present level of experimental precision, CP violation in B mixing is a negligible effect. Thus, for the purpose of analyzing CP asymmetries in hadronic B decays, we can use

$$\lambda_f = e^{-i\phi_{M(B)}}\left(\overline{A}_f/A_f\right) , \tag{1.84}$$

where $\phi_{M(B)}$ refers to the phase of $\mathbf{M}_{12}$ appearing in Eq. (1.44) that is appropriate for $B^0 - \overline{B}^0$ oscillations. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_{M(B)}} = (V^*_{tb}V_{td}) \,/\, (V_{tb}V^*_{td}) \ . \tag{1.85}$$

Some of the most interesting decays involve final states that are common to B^0 and $\overline{B}^0$ [45,46]. It is convenient to rewrite Eq. (1.42) for B decays as [47–49]

$$\begin{aligned} &\mathcal{A}_f\,(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)\,, \\ &S_f \equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1 + \left|\lambda_f\right|^2}, \quad C_f \equiv \frac{1 - \left|\lambda_f\right|^2}{1 + \left|\lambda_f\right|^2}\ , \end{aligned} \tag{1.86}$$

where we assume that $\Delta\Gamma = 0$ and $|q/p| = 1$. An alternative notation in use is $A_f \equiv -C_f$, but this A_f should not be confused with the A_f of Eq. (1.15).

A large class of interesting processes proceed via quark transitions of the form $\overline{b} \to \overline{q}q\overline{q}'$ with $q' = s$ or d. For $q = c$ or u, there are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 1.2) which carry different weak phases:

$$A_f = \left(V^*_{qb}V_{qq'}\right) t_f + \sum_{q_u = u,c,t}\left(V^*_{q_u b}V_{q_u q'}\right) p_f^{q_u} \ . \tag{1.87}$$

(The distinction between tree and penguin contributions is a heuristic one; the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, Ref. [50].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations. For example, for $f = \pi\pi$, which proceeds via $\overline{b} \to \overline{u}u\overline{d}$ transition, we can write

$$A_{\pi\pi} = (V^*_{ub}V_{ud})\,T_{\pi\pi} + (V^*_{tb}V_{td})\,P^t_{\pi\pi}, \tag{1.88}$$

where $T_{\pi\pi} = t_{\pi\pi} + p^u_{\pi\pi} - p^c_{\pi\pi}$ and $P^t_{\pi\pi} = p^t_{\pi\pi} - p^c_{\pi\pi}$. CP-violating phases in Eq. (1.88) appear only in the CKM elements, so that

$$\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{\left(V_{ub}V^*_{ud}\right) T_{\pi\pi} + \left(V_{tb}V^*_{td}\right) P^t_{\pi\pi}}{\left(V^*_{ub}V_{ud}\right) T_{\pi\pi} + \left(V^*_{tb}V_{td}\right) P^t_{\pi\pi}}. \tag{1.89}$$

For $f = J/\psi K$, which proceeds via $\overline{b} \to \overline{c}c\overline{s}$ transition, we can write

$$A_{\psi K} = (V^*_{cb}V_{cs})\, T_{\psi K} + (V^*_{ub}V_{us})\, P^u_{\psi K}, \tag{1.90}$$

where $T_{\psi K} = t_{\psi K} + p^c_{\psi K} - p^t_{\psi K}$ and $P^u_{\psi K} = p^u_{\psi K} - p^t_{\psi K}$. A subtlety arises in this decay that is related to the fact that B^0 decays into a final $J/\psi K^0$ state while $\overline{B}^0$ decays into a final $J/\psi \overline{K}^0$ state. A common final state, *e.g.*, $J/\psi K_S$, is reached only via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor (defined in Eq. (1.44)) corresponding to neutral K mixing, $e^{-i\phi_{M(K)}} = (V^*_{cd}V_{cs})/(V_{cd}V^*_{cs})$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{\left(V_{cb}V^*_{cs}\right) T_{\psi K} + \left(V_{ub}V^*_{us}\right) P^u_{\psi K}}{\left(V^*_{cb}V_{cs}\right) T_{\psi K} + \left(V^*_{ub}V_{us}\right) P^u_{\psi K}} \times \frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}. \tag{1.91}$$

For $q = s$ or d, there are only penguin contributions to A_f, that is, $t_f = 0$ in Eq. (1.87). (The tree $\overline{b} \to \overline{u}u\overline{q}'$ transition followed by $\overline{u}u \to \overline{q}q$ rescattering is included below in the P^u terms.) Again, CKM unitarity allows us to write A_f in terms of two CKM combinations. For example, for $f = \phi K_S$, which proceeds via $\overline{b} \to \overline{s}s\overline{s}$ transition, we can write

$$\frac{\overline{A}_{\phi K_S}}{A_{\phi K_S}} = -\frac{\left(V_{cb}V^*_{cs}\right) P^c_{\phi K} + \left(V_{ub}V^*_{us}\right) P^u_{\phi K}}{\left(V^*_{cb}V_{cs}\right) P^c_{\phi K} + \left(V^*_{ub}V_{us}\right) P^u_{\phi K}} \times \frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}, \tag{1.92}$$

where $P^c_{\phi K} = p^c_{\phi K} - p^t_{\phi K}$ and $P^u_{\phi K} = p^u_{\phi K} - p^t_{\phi K}$.

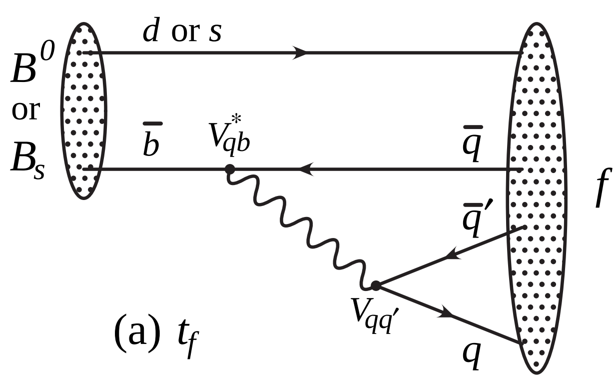
Since the amplitude A_f involves two different weak phases, the corresponding decays can exhibit both CP violation in the interference of decays with and without mixing, $S_f \neq 0$, and CP violation in decays, $C_f \neq 0$. (At the present level of experimental precision, the contribution to C_f from CP violation in mixing is negligible, see Eq. (1.82).) If the contribution from a second weak phase is suppressed, then the interpretation of S_f in terms of Lagrangian CP-violating parameters is clean, while C_f is small. If such a second contribution is not suppressed, S_f depends on hadronic parameters and, if the relevant strong phase is large, C_f is large.

A summary of $\overline{b} \to \overline{q}q\overline{q}'$ modes with $q' = s$ or d is given in Table 1.1. The $\overline{b} \to \overline{d}d\overline{q}$ transitions lead to final states that are similar to the $\overline{b} \to \overline{u}u\overline{q}$ transitions and have similar phase dependence. Final states that consist of two-vector mesons ($\psi\phi$ and $\phi\phi$) are not CP eigenstates, and angular analysis is needed to separate the CP-even from the CP-odd contributions.

The cleanliness of the theoretical interpretation of S_f can be assessed from the information in the last column of Table 1.1. In case of small uncertainties, the expression for S_f in terms of CKM phases can be deduced from the fourth column of Table 1.1 in combination with Eq. (1.85) (and, for $b \to q\overline{q}s$ decays, the example in Eq. (1.91)). Here we consider several interesting examples.

For $B \to J/\psi K_S$ and other $\overline{b} \to \overline{c}c\overline{s}$ processes, we can neglect the P^u contribution to A_f, in the Standard Model, to an approximation that is better than one percent:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0\,. \tag{1.93}$$



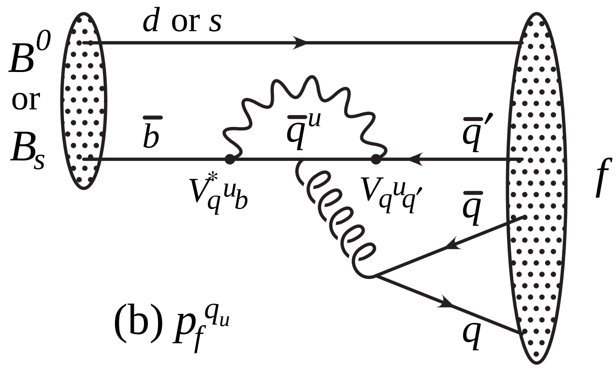


Figure 1.2: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\overline{b} \to \overline{q}q\overline{q}'$ quark-level process.

In the presence of new physics, A_f is still likely to be dominated by the T term, but the mixing amplitude might be modified. We learn that, model-independently, $C_f \approx 0$ while S_f cleanly determines the mixing phase $(\phi_M - 2\arg(V_{cb}V_{cd}^*))$. The experimental measurement [25], $S_{\psi K} = 0.665 \pm 0.022$, gave the first precision test of the Kobayashi-Maskawa mechanism, and its consistency with the predictions for $\sin 2\beta$ makes it very likely that this mechanism is indeed the dominant source of CP violation in meson decays.

For $B \to \phi K_S$ and other $\overline{b} \to \overline{s}s\overline{s}$ processes (as well as some $\overline{b} \to \overline{u}u\overline{s}$ processes), we can neglect the subdominant contributions, in the Standard Model, to an approximation that is good on the order of a few percent:

$$\lambda_{\phi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\phi K_S} = \sin 2\beta, \quad C_{\phi K_S} = 0\,. \tag{1.94}$$

In the presence of new physics, both A_f and $\mathbf{M}_{12}$ can get contributions that are comparable in size to those of the Standard Model and carry new weak phases. Such a situation gives several interesting

Table 1.1: Summary of $\overline{b} \to \overline{q}q\overline{q}'$ modes with $q' = s$ or d. The second and third columns give examples of final hadronic states. The fourth column gives the CKM dependence of the amplitude A_f, using the notation of Eqs. (1.88, 1.90,1.92), with the dominant term first and the subdominant second. The suppression factor of the second term compared to the first is given in the last column. "Loop" refers to a penguin versus tree-suppression factor (it is mode-dependent and roughly $\mathcal{O}(0.2-0.3)$) and $\lambda = 0.23$ is the expansion parameter of Eq. (1.50).

$\overline{b} \to \overline{q}q\overline{q}'$	$B^0 \to f$	$B_s \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^* V_{cs})T + (V_{ub}^* V_{us})P^u$	loop $\times\, \lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^* V_{cd})T + (V_{tb}^* V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	$K_S K_S$	ϕK_S	$(V_{tb}^* V_{td})P^t + (V_{cb}^* V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\rho^0 K_S$	$(V_{ub}^* V_{ud})T + (V_{tb}^* V_{td})P^t$	loop

consequences for penguin-dominated $b \to q\overline{q}s$ decays ($q = u, d, s$) to a final state f:

1. The value of $-\eta_f S_f$ may be different from $S_{\psi K_S}$ by more than a few percent, where η_f is the CP eigenvalue of the final state.
2. The values of $\eta_f S_f$ for different final states f may be different from each other by more than a few percent (for example, $S_{\phi K_S} \neq S_{\eta' K_S}$).
3. The value of C_f may be different from zero by more than a few percent.

While a clear interpretation of such signals in terms of Lagrangian parameters will be difficult because, under these circumstances, hadronic parameters do play a role, any of the above three options will clearly signal new physics. Fig. 1.3 summarizes the present experimental results: none of the possible signatures listed above is unambiguously established, but there is definitely still room for new physics.

For $B \to \pi\pi$ and other $\overline{b} \to \overline{u}u\overline{d}$ processes, the penguin-to-tree ratio can be estimated using SU(3) relations and experimental data on related $B \to K\pi$ decays. The result is that the suppression is on the order of $0.2 - 0.3$ and so cannot be neglected. The expressions for $S_{\pi\pi}$ and $C_{\pi\pi}$ to leading order in $R_{PT} \equiv (|V_{tb}V_{td}|\, P^t_{\pi\pi})/(|V_{ub}V_{ud}|\, T_{\pi\pi})$ are:

$$\lambda_{\pi\pi} = e^{2i\alpha}\left[\left(1 - R_{PT}e^{-i\alpha}\right) / \left(1 - R_{PT}e^{+i\alpha}\right)\right] \Rightarrow$$
$$S_{\pi\pi} \approx \sin 2\alpha + 2\,\mathcal{R}e(R_{PT})\cos 2\alpha \sin\alpha, \quad C_{\pi\pi} \approx 2\,\mathcal{I}m(R_{PT})\sin\alpha. \tag{1.95}$$

Note that R_{PT} is mode-dependent and, in particular, could be different for $\pi^+\pi^-$ and $\pi^0\pi^0$. If strong phases can be neglected, then R_{PT} is real, resulting in $C_{\pi\pi} = 0$. The size of $C_{\pi\pi}$ is an indicator of how large the strong phase is. The present experimental range is

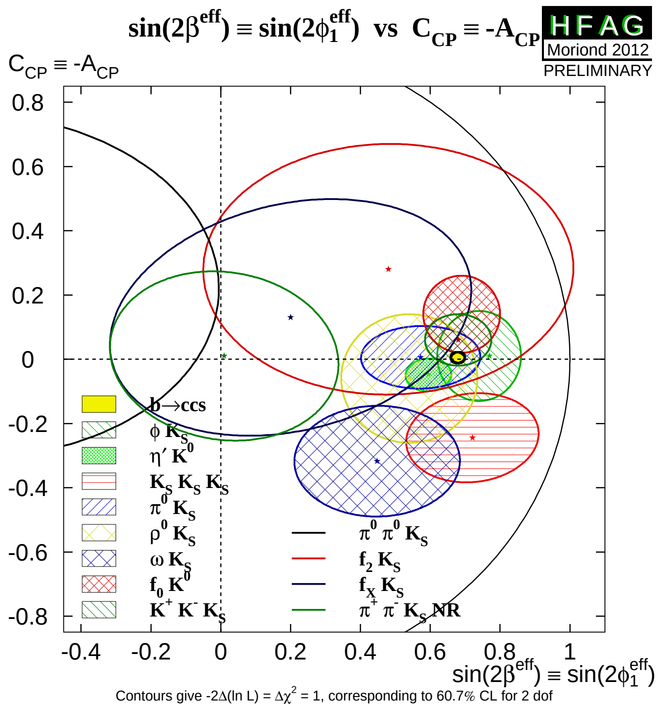


Figure 1.3: Summary of the results [25] of time-dependent analyses of $b \to q\overline{q}s$ decays, which are potentially sensitive to new physics.

$C_{\pi\pi} = -0.38 \pm 0.06$ [25]. As concerns $S_{\pi\pi}$, it is clear from Eq. (1.95) that the relative size or strong phase of the penguin contribution must be known to extract α. This is the problem of penguin pollution.

The cleanest solution involves isospin relations among the $B \to \pi\pi$ amplitudes [51]:

$$\frac{1}{\sqrt{2}} A_{\pi^+\pi^-} + A_{\pi^0\pi^0} = A_{\pi^+\pi^0}. \tag{1.96}$$

The method exploits the fact that the penguin contribution to $P^t_{\pi\pi}$ is pure $\Delta I = \frac{1}{2}$ (this is not true for the electroweak penguins which, however, are expected to be small), while the tree contribution to $T_{\pi\pi}$ contains pieces which are both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$. A simple geometric construction then allows one to find R_{PT} and extract α cleanly from $S_{\pi^+\pi^-}$. The key experimental difficulty is that one must measure accurately the separate rates for $B^0, \overline{B}^0 \to \pi^0\pi^0$.

CP asymmetries in $B \to \rho\pi$ and $B \to \rho\rho$ can also be used to determine α. In particular, the $B \to \rho\rho$ measurements are presently very significant in constraining α. The extraction proceeds via isospin analysis similar to that of $B \to \pi\pi$. There are, however, several important differences. First, due to the finite width of the ρ mesons, a final $(\rho\rho)_{I=1}$ state is possible [52]. The effect is, however, small, on the order of $(\Gamma_\rho/m_\rho)^2 \sim 0.04$. Second, due to the presence of three helicity states for the two-vector mesons, angular analysis is needed to separate the CP-even and CP-odd components. The theoretical expectation is, however, that the CP-odd component is small. This expectation is supported by experiments which find that the $\rho^+\rho^-$ and $\rho^\pm\rho^0$ modes are dominantly longitudinally polarized. Third, an important advantage of the $\rho\rho$ modes is that the penguin contribution is expected to be small due to different hadronic dynamics. This expectation is confirmed by the smallness of $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.73 \pm 0.28) \times 10^{-6}$ compared to $\mathcal{B}(B^0 \to \rho^+\rho^-) = (24.2 \pm 3.1) \times 10^{-6}$. Thus, $S_{\rho^+\rho^-}$ is not far from $\sin 2\alpha$. Finally, both $S_{\rho^0\rho^0}$ and $C_{\rho^0\rho^0}$ are experimentally accessible, which may allow a precision determination of α. The consistency between the range of α determined by the $B \to \pi\pi, \rho\pi, \rho\rho$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

An interesting class of decay modes is that of the tree level decays $B^\pm \to D^{(*)0}K^\pm$. These decays provide golden methods for a clean determination of the angle γ [53–56]. The method uses the decays $B^+ \to D^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{u}c\overline{s}$, and $B^+ \to \overline{D}^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{c}u\overline{s}$, with the D^0 and $\overline{D}^0$ decaying into a common final state. The decays into common final states, such $(\pi^0K_S)_DK^+$, involve interference effects between the two amplitudes, with sensitivity to the relative phase, $\delta + \gamma$ (δ is the relevant strong phase). The CP-conjugate processes are sensitive to $\delta - \gamma$. Measurements of branching ratios and CP asymmetries allow an extraction of γ and δ from amplitude triangle relations. The extraction suffers from discrete ambiguities but involves no hadronic uncertainties. However, the smallness of the CKM-suppressed $b \to u$ transitions makes it difficult at present to use the simplest methods [53–55] to determine γ. These difficulties are overcome (and the discrete ambiguities are removed) by performing a Dalitz plot analysis for multi-body D decays [56]. The consistency between the range of γ determined by the $B \to DK$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

The upper bound on the CP asymmetry in semileptonic B_s decays [25] implies that CP violation in $B_s - \overline{B_s}$ mixing is a small effects:

$$\mathcal{A}_{\rm SL}^s = (-10.5 \pm 6.4) \times 10^{-3} \quad \Longrightarrow \quad |q/p| = 1.0052 \pm 0.0032. \tag{1.97}$$

Neglecting the deviation of $|q/p|$ from 1, implies that we can use

$$\lambda_f = e^{-i\phi_M(B_s)} \left(\overline{A}_f/A_f\right). \tag{1.98}$$

Within the Standard Model,

$$e^{-i\phi_{M(B_s)}} = (V_{tb}^* V_{ts}) \,/\, (V_{tb} V_{ts}^*) \;. \tag{1.99}$$

Note that $\Delta\Gamma/\Gamma = 0.15 \pm 0.02$ [25] and therefore y should not be put to zero in Eqs. (1.32, 1.33). However, $|q/p| = 1$ is expected to hold to an even better approximation than for B mesons. The $B_s \to J/\psi\phi$ decay proceeds via the $b \to c\bar{c}s$ transition. The CP asymmetry in in this mode thus determines (with angular analysis to disentangle the CP-even and CP-odd components of the final state) $\sin 2\beta_s$, where β_s is defined in Eq. (1.53). The combination of CDF, D0 and LHCb measurements yields [25]

$$\beta_s = 0.07^{+0.06}_{-0.08}, \tag{1.100}$$

consistent with the Standard Model prediction, $\beta_s = 0.018 \pm 0.001$ [18].

1.7. Summary and Outlook

CP violation has been experimentally established in K and B meson decays. A full list of CP asymmetries that have been measured at a level higher than 5σ is given in the introduction to this review. In Section 1.1.4 we introduced three types of CP-violating effects. Examples of these three types include the following:

1. All three types of CP violation have been observed in $K \to \pi\pi$ decays:

$$\begin{aligned}
\mathcal{R}e(\epsilon') &= \frac{1}{6}\left(\left|\frac{\overline{A}_{\pi^0\pi^0}}{A_{\pi^0\pi^0}}\right| - \left|\frac{\overline{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}}\right|\right) = (2.5 \pm 0.4) \times 10^{-6} \quad \text{(I)} \\
\mathcal{R}e(\epsilon) &= \frac{1}{2}\left(1 - \left|\frac{q}{p}\right|\right) = (1.66 \pm 0.02) \times 10^{-3} \quad \text{(II)} \\
\mathcal{I}m(\epsilon) &= -\frac{1}{2}\mathcal{I}m(\lambda_{(\pi\pi)_{I=0}}) = (1.57 \pm 0.02) \times 10^{-3}\ . \quad \text{(III)}
\end{aligned} \tag{1.101}$$

2. Direct CP violation has been observed in, for example, the $B^0 \to K^+\pi^-$ decays, while CP violation in interference of decays with and without mixing has been observed in, for example, the $B \to J/\psi K_S$ decay:

$$\begin{aligned}
\mathcal{A}_{K^+\pi^-} &= \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.087 \pm 0.008 \quad \text{(I)} \\
S_{\psi K} &= \mathcal{I}m(\lambda_{\psi K}) = +0.679 \pm 0.020\ . \quad \text{(III)}
\end{aligned} \tag{1.102}$$

Based on Standard Model predictions, further observation of CP violation in D, B and B_s decays seems promising for the near future, at both LHCb and a possible higher-luminosity asymmetric-energy B factory [58]. Observables that are subject to clean theoretical interpretation, such as $S_{\psi K_S}$, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ and CP violation in $B \to DK$ decays, are of particular value for constraining the values of the CKM parameters and probing the flavor sector of extensions to the Standard Model. Other probes of CP violation now being pursued experimentally include the electric dipole moments of the neutron and electron, and the decays of tau leptons. Additional processes that are likely to play an important role in future CP studies include top-quark production and decay, and neutrino oscillations.

All measurements of CP violation to date are consistent with the predictions of the Kobayashi-Maskawa mechanism of the Standard Model. Actually, it is now established that the KM mechanism plays a major role in the CP violation measured in meson decays. However, a dynamically-generated matter-antimatter asymmetry of the universe requires additional sources of CP violation, and such sources are naturally generated by extensions to the Standard Model. New sources might eventually reveal themselves as small deviations from the predictions of the KM mechanism in meson decay rates, or else might not be observable in meson decays at all, but observable with future probes such as neutrino oscillations or electric dipole moments. We cannot guarantee that new sources of CP violation will ever be found experimentally, but the fundamental nature of CP violation demands a vigorous effort.

A number of excellent reviews of CP violation are available [59–67], where the interested reader may find a detailed discussion of the various topics that are briefly reviewed here.

We thank Tim Gershon for significant contributions to the 2012 update.

Further discussion and all references may be found in the full *Review of Particle Physics.*